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DESIGN OF DISTRIBUTED ENGINE CONTROL SYSTEMS FOR STABILITY UNDER COMMUNICATION PACKET DROPOUTS (POSTPRINT)

Rama K. Yedavalli, Rohit K. Belapurkar, and Alireza R. Behbahani

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Rama K. Yedavalli and Rohit K. Belapurkar (The Ohio State University)
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The Ohio State University
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Fan and Compressor Branch (AFRL/RZTF)
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Air Force Research Laboratory, Propulsion Directorate
Wright-Patterson Air Force Base, OH 45433-7251
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14. ABSTRACT
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Design of Distributed Engine Control Systems for Stability Under Communication Packet Dropouts

Rama K. Yedavalli[*] and Rohit K. Belapurkar[†]
The Ohio State University, Columbus, Ohio, 43210, USA

Alireza Behbahani[‡]
Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433, USA

In this paper, we address the issue of stability of Distributed Engine Control Systems under communication constraints and in particular for packet dropouts. We propose a control design procedure labeled Decentralized Distributed Full Authority Digital Engine Control (D^{2}FADEC) based on a two level decentralized control framework. We show that, Packet Dropping Margin (PDM), which is a measure of stability robustness under packet dropouts, is largely dependent on the closed loop controller structure; and that in particular block diagonal structure is more desirable. Thus, we design a controller in a decentralized framework to improve the PDM. The effect of different mathematical partitioning on the PDM is studied. The proposed methodology is applied to a F100 gas turbine engine model which clearly demonstrates the usefulness of decentralization in improving the stability of distributed control under packet dropouts.

Nomenclature

ξ = Independent and Identically Distributed (i.i.d.) Bernoulli random process
$\rho(\cdot)$ = Spectral radius of a matrix
$\otimes$ = Kronecker product
μ = λ_{max}^+, where λ^+ are eigenvalues having positive real part
$\|\cdot\|_2$ = Spectral norm of a matrix
$K_2(\cdot)$ = Spectral condition number of a matrix
$(\cdot)^{\#}$ = Moore-Penrose Inverse of a matrix

I.   Introduction

In recent years, increasingly sophisticated electronics have been added to the engine control system for addressing the needs of increased performance, wider operability, and reduced life-cycle cost. Future engines are expected to have higher engine thrust-to-weight ratio, low engine fuel consumption and low overall engine cost (see, for instance, [1]). Research is being carried out to make aircraft propulsions systems more intelligent, reliable, self-diagnostic, self-prognostic, self-optimizing and mission adaptable while also reducing the engine acquisition and maintenance cost. This has driven the need for a new, advanced control system. Accordingly, a working group was formed to study and develop a new Distributed Engine Control (DEC)[2-3]. The advantages of decentralized control scheme for gas turbine engine are also well discussed in literature. [4-6]. In this

paper, we extend the decentralized scheme to distributed control and propose a new framework labeled Decentralized Distributed Full Authority Digital Engine Control (D^{2}FADEC). Towards this direction, we address the issue of stability under packet dropouts and review the concept of Packet Dropout Margin (PDM), which is a measure of stability robustness under packet dropouts (See, for instance, [7]). Hu and Yan designed a controller based on a centralized framework to improve the PDM. In this paper, we show that PDM is dependent on a closed loop system matrix structure and demonstrate that controllers designed based on a decentralized framework further improve the PDM with the same nominal performance as the centralized controller. The paper is organized as follows. In section II, we briefly summarize the distributed engine control systems literature along with a discussion on communications constraints in Networked Control Systems (NCS). In section III, we address the issue of packet dropouts in networked control systems and review the concept of PDM introduced by Hu and Yan. In section IV, a mathematical formulation is developed to show that controllers designed in decentralized framework improve the PDM significantly compared to centralized framework controllers. In addition, the effect of mathematical partitioning on PDM is studied. In section V, we propose a new framework based on decentralization for distributed engine control systems labeled Decentralized Distributed Full Authority Digital Engine Control (D^{2}FADEC). Finally section VI offers concluding remarks.

II.   Distributed Engine Control Systems
A. FADEC based on Distributed Engine Control Architecture (DEC)

In Distributed Engine Control, the functions of Full Authority Digital Engine Control (FADEC) are distributed at the component level. Each sensor/actuator is to be replaced by a smart sensor/actuator. These smart modules include local processing capability to allow modular signal acquisition and conditioning, and diagnostics and health management functionality. Dual channel digital serial communication network is used to connect these smart modules with FADEC. Fig. 1 shows the schematic of FADEC based on distributed control architecture.

[*]Professor, Department of Aerospace Engineering; yedavalli.1@osu.edu. Associate Fellow AIAA.
[†]Doctoral Student, Department of Aerospace Engineering; belapurkar.2@osu.edu. Student Member AIAA.
[‡]Senior Aerospace Engineer, Air Force Research Laboratory; al.behbahani@wpafb.af.mil. Senior Member AIAA.
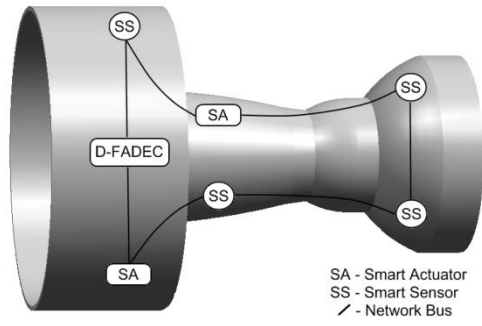
Fig. 1 FADEC based on Distributed architecture

Reduction of engine control system weight, modularity, obsolescence reduction, scalability, and reduction in operational and maintenance cost are some of the perceived benefits of DEC. (See, for instance, [8-9]). The distributed control approach is inherently more powerful, flexible, and scalable than a centralized control approach. However, there are major technical challenges to the realization of DEC. High temperature electronics, selection of appropriate communication architecture, and partitioning of the centralized controller are some to name a few. As the performance of the DEC will be dependent on the performance of the communication network, the appropriate selection of communication architecture is very important.

In order to reduce the development time and cost, use of the commercial off-the-shelf communication architecture is preferred. Hence, we first select the communication architecture desired for the distributed system, and then partition the controller, given the communication constraints, to improve system's performance under these constraints.

B. Selection of Communication Architectures

For safety-critical DCS, there is a clear preference for time-triggered protocols over the event-driven protocols. Time-triggered protocols offer high level of reliability with fault-tolerance. These architectures ensure that maximum bus loading stays at prescribed levels and also provides efficiency, determinism and partitioning. Some of the existing off-the-shelf open system communication standards are MIL-STD-1553, SAFEbus, FlexRay, CAN, SPIDER, TTTech Time-Triggered Architecture (TTA), and IEEE 1394b/Firewire. Honeywell SAFEbus is used in the Boeing 777 Airplane Information Management System, TTP/C of TTTech Computertechnik AG is used in the environmental control systems in Boeing 787 Dreamliner as well as cabin pressure control systems in the Airbus 380 and IEEE 1394b is under development for use in Joint Strike Fighter (JSF). Out of these architectures, TTP/C has clear advantages over the others (see, for instance, [10-12]). Some of the requirements of communication architecture for DCS are that it should support fault detection, isolation, and recovery (FDIR) and health onitoring; should be highly modular, with high reliability, should be easy to maintain and finally should have low overall cost. All these requirements are best met by TTP/C. TTP/C is specially designed for the safety critical, hard real-time distributed control. Along with high transmission rate, TTP/C has high data efficiency, error detection with short latency, a fault-tolerant clock synchronization service, and distributed redundancy management. This architecture can tolerate multiple faults and high degree of temporal predictability. TTP/C provides support for a fiber-optics physical layer as well as for an electrical

physical layer. In the next section, we review some of the features of the communication architecture which are relevant for DEC.

C. Networked Control Systems (NCS)

Distributed Engine Control Systems can be viewed as a Networked Control System (NCS) with distributed sensors and actuators. Here, the control loops are closed through a real-time communication network. There are various factors introduced as a result of addition of the communication network. They include network induced time delay, packet dropouts, and bandwidth constraints, which have to be considered for ensuring desired functionality of the NCS [13-16]

1. Network-induced Time Delay

Time delays occur in networked control system due to the addition of a network. This delay can destabilize the system designed without considering the delay or can degrade the system performance. Networked induced delay can be further subdivided into sensor-to-controller delay, controller-to-actuator delay and the computational delay in the controller. In the selected TTP/C architecture, use of clock synchronization, transmission window timing and group membership ensure that the time delays are constant and bounded [17].

2. Constraint on Channel Bandwidth

The capacity of the communication network to carry a finite amount of information per unit amount of time is known as channel bandwidth. The current available TTP/C hardware supports 25 Mbit/s synchronous and 5 Mbit/s asynchronous transmissions. The actual available bandwidth is determined by the physical layer of the network. We consider the use of Fiber Optic physical layer which would enable data transmission at high speeds with immunity to Electro Magnetic Interference (EMI).

3. Packet Dropouts

Packet collisions or node failures can result in loss of information packets, which is known as packet dropouts. In time triggered protocol, time division multiple access (TDMA) mechanism ensures that each node can transmit data only during the predetermined time slot thereby reducing the likelihood of packet dropouts due to packet collisions. However, the network is still subjected to node failures. When a node failure occurs, instead of repeating retransmission attempts, it is advantageous to drop the old packet and transmit a new one.

The membership mechanism of TTP is capable of detecting any kind of communication fault that is not already detected and handled by other means. These communication faults include transmission and reception faults. If a node fails to transmit, which is typically due to noise during the transmission, it is removed from the membership list and is not allowed to transmit data. Immediate retransmission for this node is not allowed and it can retry transmission in the next round [17]. Also, if the packet fails the cyclic redundancy check (CRC), the packet is dropped and the transmitting node is required to wait for its next TDMA cycle in order to transmit another message. *Hence, for communication architectures implemented using TTP/C, it is important to consider stability and performance of the system under packet dropouts.* In this paper, we analyze the effect of packet dropouts on the stability of the system considering single-packet transmission of plant inputs and outputs.

III. Stability of Networked Control Systems under Packet Dropouts

The packet dropouts in a communication network can be modeled as either an independent and identically distributed (i.i.d.)

Bernoulli random process or a Markov chain. Hu and Yan studied the effect of packet dropping in [7]. The packet dropouts in a communication network were modeled as an i.i.d Bernoulli process and the stability of a discrete-time NCS with static state feedback was studied. A formula for calculation of Packet Dropping Margin (PDM), an upper bound on the Packet Dropping Probability (PDP), which guarantees system stability, was derived. Stability of networked control systems under packet dropouts is briefly summarized below.
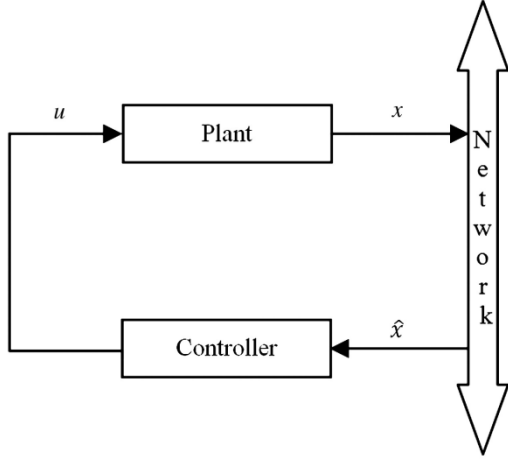

Fig. 2 Networked Control Systems with packet dropouts

Consider a networked control system as shown in Figure 2. The network is assumed to be modeled by
$$\hat{x}(k) = \xi(k)\, x(k) \qquad (1)$$
where, $\xi(k)$ is the independent and identically distributed (i.i.d.) Bernoulli random process. $\xi(k)$ can be either 0 or 1 at any time instant k. Value 0 indicates the packet is lost during transmission while value 1 indicates successful transmission of packet. The probability of $\xi(k) = 0$ is termed as Packet Dropping Probability (PDP) and is a measure of the reliability of the network.

Now, when the packet is dropped, $\xi(k) = 0$ i.e., $\hat{x}(k) = 0$. If the packet is transmitted successfully $\xi(k) = 1$ and $\hat{x}(k) = x(k)$.

Stability Condition for NCS: The networked plant with PDP equal to the constant α is ms-stabilized (stabilized in mean-square sense) by the controller if and only if the following condition holds [18]:
$$\rho\left[\alpha A \otimes A + (1-\alpha)\hat{A} \otimes \hat{A}\right] < 1 \qquad (2)$$
where, $\otimes$ is the Kronecker product and $\rho(\cdot)$ is the spectral radius of the matrix and $\hat{A} = A - BK$

Hu and Yan introduced a term known as Packet Dropping Margin (PDM) which is defined as the largest positive bound α_{max} such that the system is ms-stable for any PDP less than α_{max}. A formula for calculating PDM is as follows [7]:
If the NCS is nominally stable, then,
$$PDM = \frac{1}{\mu(V)}$$
where, $\qquad (3)$
$$V = \begin{bmatrix} (S \otimes \tilde{S} + \tilde{S} \otimes S)(I - S \otimes S)^{-1} & \tilde{S} \otimes \tilde{S} \\ (I - S \otimes S)^{-1} & 0 \end{bmatrix} \qquad (4)$$
$$S = \hat{A} \otimes \hat{A}, \qquad \tilde{S} = A \otimes A - \hat{A} \otimes \hat{A}$$

Lower bound for PDM, which is dependent on the $K_2(\hat{A})$, is given by the following equation.

$$PDM \geq \frac{1-\rho^2(\hat{A})}{K_2^2(\hat{A})\|A\|_2^2 - \rho^2(\hat{A})}$$
$\qquad (5)$

From the above equation, it is observed that PDM is inversely proportional to $K_2(\hat{A})$. Hence, to maximize the PDM, Hu and Yan proposed using a robust pole placement technique, which minimizes $K_2(\hat{A})$ using an ODE-based algorithm [7].

IV. Decentralized Controller Design for PDM Improvement

The above algorithm is computationally expensive and hence it is important to find a method to increase PDM using a less computationally expensive method. In this paper, we offer a solution to improve the PDM by exploiting the structural properties of block diagonal matrices in comparison to non block diagonal matrices. In particular, we recall the following theorem that explicitly gives a relation between the structure of a matrix and condition number of the matrix.

Theorem: Let A_T and A_D be a block triangular and block diagonal matrix respectively given by
$$A_T = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}, \quad A_D = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$$
Then,
$$K_2(A_D) \leq K_2(A_T)$$
Proof: We know that the condition number is given as,
$$K_2(\cdot) = \frac{\sigma_{max}(\cdot)}{\sigma_{min}(\cdot)}$$
And from Theorem I given in Ref. [19],
$$\sigma_{max}(A_D) \leq \sigma_{max}(A_T)$$
$$\sigma_{min}(A_D) \geq \sigma_{min}(A_T)$$
Hence,
$$K_2(A_D) \leq K_2(A_T)$$

Example 1: Consider a linear state space system in discrete time framework, with system matrices shown below,
$$A = \begin{bmatrix} 1.2 & -0.3 & 0.6 & -0.1 \\ 0.4 & 0.1 & -0.4 & 0.9 \\ -0.5 & 1.5 & 0.3 & 0.4 \\ 0.6 & -0.3 & 0.7 & -0.9 \end{bmatrix}$$
$$B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -0.4 & 0 & 0 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$
Suppose, the desired nominal closed loops are -0.052, -0.4131, 0.758, and 0.897

The closed loop matrix $\hat{A}$, having above eigenvalues can be obtained using gain matrix K_T and K_D as shown, where $\hat{A}_T$ is the closed loop system matrix with controller K_T and $\hat{A}_D$ is the closed loop system matrix with gain K_D.
$$K_T = \begin{bmatrix} 1.0889 & -0.7087 & 0.4826 & -0.4188 \\ 0.3548 & -1.2373 & -1.7418 & 1.1895 \\ 0.7143 & -2.1429 & -0.0540 & 0.4446 \\ -1.2 & 0.6 & -0.1944 & 2.2434 \end{bmatrix}$$
$$K_D = \begin{bmatrix} 1.0889 & -0.7087 & 0.6 & -0.1 \\ 0.3548 & -1.2373 & -1 & 2.25 \\ 0.7143 & -2.1429 & -0.054 & 0.4446 \\ -1.2 & 0.6 & -0.1944 & 2.2434 \end{bmatrix}$$
$$\hat{A}_T = \begin{bmatrix} 0.1111 & 0.4087 & 0.1174 & 0.3188 \\ 0.2581 & 0.5949 & 0.2967 & 0.424 \\ 0 & 0 & 0.2622 & 0.7112 \\ 0 & 0 & 0.6028 & 0.2217 \end{bmatrix}$$

$$\hat{A}_D = \begin{bmatrix} 0.1111 & 0.4087 & 0 & 0 \\ 0.2581 & 0.5949 & 0 & 0 \\ 0 & 0 & 0.2622 & 0.7112 \\ 0 & 0 & 0.6028 & 0.2217 \end{bmatrix}$$

Note that $\hat{A}_T$ is a block triangular matrix, whereas $\hat{A}_D$ is a block diagonal matrix, both having the same eigenvalues.

It is observed that $K_2(\hat{A}_T) = 24.0274$ and $K_2(\hat{A}_D) = 17.7659$, which in turn give

$$PDM_T = 0.0832 \quad PDM_D = 0.5313$$

Thus, as declared in the theorem, the block diagonal structure results in lower condition number and also produces higher PDM. Hence, we observe that the PDM is largely dependent on the structure of the closed loop matrix and by having a block diagonal closed loop matrix, we can increase PDM significantly. Encouraged and motivated by this observation, in what follows, we propose a decentralized controller scheme which generates a block diagonal closed loop matrix, thereby improving the PDM. Furthermore, we study the effect of partitioning in the decentralized scheme on the PDM. This is illustrated with an application in engine control.

A. Controller design procedure for Interconnected Systems for Stability Robustness under Packet Dropouts

Consider a linear system consisting of N interconnected subsystems

$$\mathbf{S:} \qquad \dot{x}_{(k+1)} = Ax_k + Bu_k \qquad (6)$$
$$y_{(k+1)} = Cx_k$$

For simplicity, we ignore the subscripts, and partition the system as-

$$\mathbf{S:} \quad \dot{x}_i = A_i x_i + B_i u_i + \sum_{j=1}^{N}(A_{ij}x_j + B_{ij}u_j) \qquad (7)$$

$$y_i = C_i x_i + \sum_{j=1}^{N} C_{ij}x_j \qquad i \in N$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^{l_i}$ are the state, input and output of subsystems, S_i

A more compact notation for the above system is given as [20]

$$\mathbf{S:} \quad \dot{x} = A_D x + B_D u + A_C x + B_C u \qquad (8)$$
$$y = C_D x + C_C x \qquad i \in N$$

where,

$$A_D = diag\{A_{D1}, A_{D2}, \dots, A_{Dn}\}$$
$$B_D = diag\{B_{D1}, B_{D2}, \dots, B_{Dm}\}$$
$$C_D = diag\{C_{D1}, C_{D2}, \dots, C_{Dl}\}$$

and coupling block matrices are

$$A_C = (A_{ij}), \qquad B_C = (B_{ij}), \qquad C_C = (C_{ij})$$

The control law for the system is given as

$$u = -Ky \qquad (9)$$

The two level controller is given as

$$u = u^l + u^g$$

The gain K can be decomposed into local and global controller gains as follows:

$$K = K_D + K_C$$

Assuming full state feedback ($C_D = C_C = I$) , the closed loop system becomes,

$$\widehat{\mathbf{S}}: \quad \dot{x} = (A_D - B_D K_D - B_C K_D)x + (A_C - B_C K_C - B_D K_C)x \quad (10)$$

$$\widehat{\mathbf{S}}: \quad \dot{x} = \hat{A}_D x + \hat{A}_C x$$

where,

$$\hat{A}_D = (A_D - B_D K_D - B_C K_D)$$
$$\hat{A}_C = (A_C - B_C K_C - B_D K_C)$$

1. Designing the local controller, K_D

Now we consider the selection of local controller gains to exponentially stabilize the overall system to prescribed degree. For local controller design, we ignore the interactions between the subsystems, i.e. $A_C = 0$. The local controller gain can be found by implementing any controller design method, such as a pole placement controller design or an optimal controller design.

2. Designing the global controller, K_C

We select global gain matrix, K_C, such that $\hat{A}_C \triangleq 0$, which corresponds to reducing the effect of the interconnections[21]. This can be done by selecting

$$K_C = B^{\#} A_C \qquad (11)$$

If matrix B is a square, non-singular matrix, then the interactions are completely nullified and $\hat{A}_C = 0$. If B is a rectangular matrix, then $\hat{A}_C \cong 0$ as B^{-1} does not exist and we have to ensure that the closed loop system remains stable. For this, we consider $\hat{A}_C$ as unstructured perturbation matrix and use the results obtained in [22] to determine system stability. This stability condition is given by

$$\sigma_{max}(\hat{A}_C) < -\sigma_{max}(\hat{A}_D) + \left([\sigma_{max}(\hat{A}_D)]^2 + \frac{\sigma_{min}(Q)}{\sigma_{max}(P)}\right)^{1/2} (12)$$

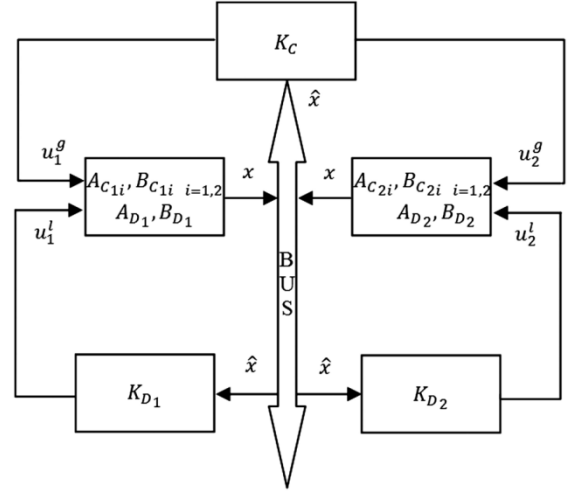Where, $Q = I$ and P is solution of discrete time Lyapunov Equation solved for $\hat{A}_D$

Fig. 3 D^{2}FADEC with distributed Local and Global Controller

Consider architecture as shown in Fig. 3. It shows a system having two subsystems, in which the local controller and global controller are connected to the subsystems using the communication network. Two types of system are now studied; one in which each subsystem has independent control ($B_C = 0$) and one where each control input affects two or more subsystems ($B_C \neq 0$).

1. Case I: $B_C = 0$

When $B_C = 0$, closed loop system reduces to following form

$$\widehat{\mathbf{S}}: \quad \dot{x} = (A_D - B_D K_D)x + (A_C - B_D K_C)x \qquad (13)$$

This can be written in a compact form as follows:

$$\widehat{\mathbf{S}}: \quad \dot{x} = \hat{A}_D x + \hat{A}_C x$$

In order to reduce the effect of interactions, matrix $\hat{A}_C$ is made 0 by the following selection of K_C.

$$K_C = B_D^{\#} A_C \qquad (14)$$

Example 2: In order to compare decentralized and centralized controllers, from PDM point of view, an example available in literature [23] is studied.

4

$$A = \begin{bmatrix} 1.2 & 0.1 & -0.3 \\ 0.5 & -0.2 & -0.3 \\ -2.5 & 1.8 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The desired nominal closed loops are -0.7053, 0.4777±0.4535i An algorithm proposed by Hu and Yan was used to find a feedback gain which maximizes PDM. This gain yields a PDM of 0.3449 and $K_2(\hat{A}) = 1.0709$

We now build a decentralized controller using the above proposed method, which yields,

$$K_D = \begin{bmatrix} 0.7223 & 0.5535 & 0 \\ 0.0930 & -1.3554 & 0 \\ 0 & 0 & 0.7053 \end{bmatrix}$$

$$K_C = \begin{bmatrix} 0 & 0 & -0.3 \\ 0 & 0 & -0.6 \\ -2.5 & 1.8 & 0 \end{bmatrix}$$

This gain yields $PDM_{DS} = 0.39$, $K_2(\hat{A}) = 1.0708$

Note that a significant improvement in PDM for the same nominal closed loop poles is observed, which confirms our previous assertion that PDM is dependent on the structure of $\hat{A}$. We also observe that $K_2(\hat{A})$, obtained with the use of a controller in decentralized framework, is lower than $K_2(\hat{A})$ obtained by Hu and Yan algorithm. As we no longer have to solve the minimization problem, the computational effort and time is significantly reduced. Hence, use of our proposed decentralized controller gives a lower $K_2(\hat{A})$, with higher PDM and with less computational effort.

2. Case II: $B_C \neq 0$
For this case, the closed loop system becomes,

$\hat{S}$: $\dot{x} = (A_D - B_D K_D)x + (A_C - B_C K_D - B_C K_C - B_D K_C)x$ (15)

$\dot{x} = (A_D - B_D K_D)x + (\widetilde{A_C} - B K_C)x$

where, $\widetilde{A_C} = A_C - B_C K_D$
or in compact form,

$\hat{S}$: $\dot{x} = \hat{A}_D x + \hat{A}_C x$

In order to reduce the effect of interactions, we make $\hat{A}_C \cong 0$ by selecting

$$K_C = B^\# \widetilde{A_C} \tag{16}$$

Example 3:
We now study the effect of packet dropouts for a F100 engine model under decentralized framework. The model is obtained from [24]. The continuous time model is converted into a discrete time model with sampling time of 0.01 s.
Let the networked plant be as shown below.

$A =$
$$\begin{bmatrix} 0.9598 & 0.0365 & -4.6317 & 0.0608 & 0.0482 & -0.0332 \\ 0.0003 & 0.9708 & -0.5745 & 0.0012 & 0.0199 & 0.1221 \\ 0.0003 & 0.0001 & 0.9556 & 0.0000 & 0.0014 & 0.0010 \\ 0.0085 & -0.0204 & -2.5621 & 0.6065 & -0.0057 & -0.0920 \\ 0.0004 & -0.0009 & -0.1158 & -0.0156 & 0.9799 & -0.0042 \\ 0.0000 & 0.0000 & -0.0059 & 0.0000 & 0.0001 & 0.9934 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0108 & 1.6642 & -0.9764 & 0.0851 \\ 0.0091 & -0.3464 & -0.0142 & -0.5932 \\ -0.0001 & -0.7790 & 0.0116 & 0.0036 \\ 0.0444 & 0.2358 & -0.1646 & 0.3287 \\ 0.0020 & 0.0103 & -0.0074 & 0.0148 \\ 0.0001 & -0.0008 & -0.0003 & 0.0007 \end{bmatrix}$$

$$C = I_{6\times6}$$
$$D = 0$$

The above networked plant was decomposed into two subsystems and the effect of mathematical partitioning on the PDM was studied. The local controller gain, K_D was obtained using pole placement technique for desired closed loop poles $0.02 \pm 0.5i$, 0.2, 0.7, 0.9801 and 0.9936. Global controller, K_C was calculated using Eq. (16).

Table 1 Dependence of PDM on the system partitioning

Type of Partitioning	PDM	$K_2(\hat{A})$
Centralized architecture	0.1955	4.9792
$A_1^{2\times2}, B_1^{2\times1}, A_2^{4\times4}, B_2^{4\times3}$	0.0117	5.476e+4
$A_1^{2\times2}, B_1^{2\times2}, A_2^{4\times4}, B_2^{4\times2}$	1.0386	4.9759
$A_1^{2\times2}, B_1^{2\times3}, A_2^{4\times4}, B_2^{4\times1}$	1.0394	2.083e+4
$A_1^{3\times3}, B_1^{3\times1}, A_2^{3\times3}, B_2^{3\times3}$	1.114e-7	2.52e+10
$A_1^{3\times3}, B_1^{3\times2}, A_2^{3\times3}, B_2^{3\times2}$	0.3204	4.9717
$A_1^{3\times3}, B_1^{3\times3}, A_2^{3\times3}, B_2^{3\times1}$	1.0338	4.9688
$A_1^{4\times4}, B_1^{4\times1}, A_2^{2\times2}, B_2^{2\times3}$	3.727e-8	3.537e+9
$A_1^{4\times4}, B_1^{4\times2}, A_2^{2\times2}, B_2^{2\times2}$	0.3285	77.4584
$A_1^{4\times4}, B_1^{4\times3}, A_2^{2\times2}, B_2^{2\times1}$	0.3835	35.7265

From the above table, it was observed that centralized controller gave a PDM of 0.1955. We also observe that the PDM depends on the system partitioning and by selecting a suitable system partition, we can obtain a large PDM. For the above example, we select partition given as $A_1^{2\times2}, B_1^{2\times2}, A_2^{4\times4}, B_2^{4\times2}$ since it gives largest PDM with small $K_2(\hat{A})$. As the PDM, which is the bound on the PDP, is more than 1, it ensures system stability for all values of PDP less than 1.

V. Decentralized Distributed Full Authority Engine Control

It has been shown that the use of decentralized control structure not only improves the performance of gas turbine engine, but also reduces the number of controller design operating points [5]. Also the controller is made more robust and the system remains stable in presence of soft and/or hard failures. Controllers based on the decentralized framework allow us to consider the interactions between the subsystems and at the same time to optimize subsystem performance. This approach provides improved component fault-prognostics and fault-tolerance while reducing the processing complexity.

In addition, for distributed engine control systems, it can now be said that a decentralized controller design as presented in this paper will also impart stability robustness with respect to packet dropouts. Hence, for distributed engine control systems, the contribution of this paper (decentralized controller design) enhances the applicability significantly.

VI. Conclusions

Advanced future propulsion control demands for an intelligent, fault tolerant systems which necessitates new control system development. The benefits of Distributed Control Systems are beginning to be recognized in the engine community. In this paper, use of TTP/C as a communication architecture is highlighted. *Decentralized Distributed Full Authority Engine Control (D^2FADEC)* is proposed and a mathematical model

consisting of two-level controller structure is analyzed for performance under packet dropouts. It is shown that the PDM is dependent on the structure of the closed-loop matrix; reducing the effect of interactions can therefore result in a significant improvement of PDM. We also demonstrate that PDM is less dependent on condition number and more dependent on the subsystem interactions. A F100 engine model, available in literature, is used to show that by selecting a suitable mathematical partitioning, we can obtain a large PDM, given that the system has prescribed nominal closed-loop poles. The same results can be extended to the case where the control input is also subject to packet dropouts.

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